

# Joint Uplink-Downlink Cell Associations for Interference Networks with Local Connectivity

Manik Singhal and Aly El Gamal  
ECE Department, Purdue University  
Email: {msingha, elgamala}@purdue.edu

**Abstract**—We study information theoretic models of interference networks that consist of  $K$  Base Station (BS) - Mobile Terminal (MT) pairs. Each BS is connected to the MT carrying the same index as well as  $L$  following MTs, where the connectivity parameter  $L \geq 1$ . We fix the value of  $L$  and study large networks as  $K$  goes to infinity. We assume that each MT can be associated with  $N_c$  BSs, and these associations are determined by a cloud-based controller that has a global view of the network. An MT has to be associated with a BS, in order for the BS to transmit its message in the downlink, or decode its message in the uplink. In previous work, the cell associations that maximize the average uplink-downlink per user degrees of freedom (puDoF) were identified for the case when  $L = 1$ . Further, when only the downlink is considered, the problem was settled for all values of  $L$  when we are restricted to use only zero-forcing interference cancellation schemes. In this work, we first propose puDoF inner bounds for arbitrary values of  $L$  when only the uplink is considered, and characterize the uplink puDoF value when  $N_c \geq L$ . We then introduce new achievable average uplink-downlink puDoF, and conjecture that the new scheme is optimal for all values of  $L$ , when we restrict our attention to zero-forcing schemes.

## I. INTRODUCTION

The fifth generation of cellular networks is expected to bring new paradigms to wireless communications, that exploit recent technological advancements like cloud computing and cooperative communication (also known as Coordinated Multi-Point or CoMP). In particular, the rising interest in Cloud Radio Access Networks (CRAN) (see e.g., [1]-[6]) holds a promise for such new paradigms. These paradigms require new information theoretic frameworks to identify fundamental limits and suggest insights that are backed by rigorous analysis. The focus of this work is to identify associations between cell edge mobile terminals and base stations, that maximize the average rate across both uplink and downlink sessions, while allowing for associating one mobile terminal with more than one base station and using cooperative transmission and reception schemes between base stations in the downlink and uplink sessions, respectively. With a cloud-based controller, optimal decisions for these associations can take into account the whole network topology, with the goal of maximizing a sum rate function.

Cloud-based CoMP communication is a promising new technology that could significantly enhance the rates of cell edge users (see [7] for an overview of CoMP). In [8], an information theoretic model was studied where cooperation was allowed between transmitters, as well as between receivers

(CoMP transmission and reception). CoMP transmission and reception in cellular networks are applicable in the downlink and uplink, respectively. The model in [8] assumed that each message can be available at  $M_t$  transmitters and can be decoded through  $M_r$  received signals. It was shown that full Degrees of Freedom (DoF) can be achieved if  $M_t + M_r \geq K + 1$ , where  $K$  is the number of transmitter-receiver pairs (users) in the network.

Recently in [9], alternative frameworks for cooperation in both downlink and uplink were introduced. The new frameworks are based on the concept of *message passing* between base stations. In the downlink, quantized versions of the analog transmit signals are being shared between base station transmitters. The supporting key idea is that information about multiple messages can be shared from one transmitter to another with the cost of sharing only one whole message (of the order of  $\log P$ , where  $P$  is the transmit power), if we only share information needed to cancel the interference caused by the messages at unintended receivers, through dirty paper coding (see [10]). In the uplink, decoded messages are shared from one base station receiver to another, where they are used to cancel interference. It was shown in [9] that there is a duality in this framework between schemes that are used in the downlink and those that are used for the uplink, with the clear advantage that the same backhaul infrastructure can be used to support both scenarios.

In this work, we study locally connected networks, where the downlink has a similar model to the one in [8] that allows each message to be available at a specified number of transmitters, and the uplink has a similar model to the one in [9] that allows sharing of digital decoded messages between receivers. We assume that in both downlink and uplink, the backhaul carries digital messages between a centralized controller and base stations. The message passing between base station receivers in the uplink can be implemented by first communicating the decoded message from its destination to the centralized controller, and then communicating the message from the centralized controller to the base station(s) that will use it to cancel interference. We impose a constraint that each message can be available at a specified maximum number of base stations. Our constraint is used to capture the backhaul rate, as well as the overhead needed to associate a mobile terminal with a base station. The justification for our choice is that sharing of analog signals face practical constraints because of quantization errors, and may not fit well

in a wireless digital infrastructure where digital messages from multiple sessions could be combined and shared over the same backhaul link.

When considering this work, it is important to note that the assumptions in a theoretical framework need not reflect directly a practical setting, but are rather used to define a tractable problem whose solution can lead to constructive insights. For example, it was shown in [11] that imposing a downlink backhaul constraint where each message can be available at a specified maximum number of transmitters (maximum transmit set size constraint), can lead to solutions that are also useful to solve the more difficult and more relevant to practice problem, where an average transmit set size constraint is used instead of the maximum. Also, in [12], it was shown that solutions obtained for the locally connected network models, that are considered in this work, can be used to obtain solutions for the more practical cellular network models, by viewing the cellular network as a set of interfering locally connected subnetworks and designing a fractional reuse scheme that avoids interference across subnetworks.

#### A. Prior Work

In [13], the considered problem was studied for Wyner's linear interference networks (channel model was introduced in [14]). The optimal message assignment and puDoF value were characterized. Linear networks form the special case of our problem when  $L = 1$ . Also, in [11], the downlink part of our problem was considered, and the optimal message assignment (cell association) and puDoF value were characterized for general values of the connectivity parameter  $L$ , when we restrict our attention to zero-forcing (or interference avoidance) scheme. Here, all our results are for general values of the connectivity parameter  $L$ .

#### B. Document Organization

In Section II, we present the problem setup. In Section III, we discuss previous work on zero-forcing CoMP transmission schemes for the downlink. We then present inner bounds for the puDoF of the uplink in Section IV, and prove the converse for a special case in Section V. In Section VI, we present new achievable puDoF values when the average of the uplink and downlink is considered. We finally present concluding remarks in Section VII.

### II. SYSTEM MODEL AND NOTATION

For each of the downlink and uplink sessions, we use the standard model for the  $K$ -user interference channel with single-antenna transmitters and receivers,

$$Y_i(t) = \sum_{j=1}^K H_{i,j}(t)X_j(t) + Z_i(t), \quad (1)$$

where  $t$  is the time index,  $X_j(t)$  is the transmitted signal of transmitter  $j$ ,  $Y_i(t)$  is the received signal at receiver  $i$ ,  $Z_i(t)$  is the zero mean unit variance Gaussian noise at receiver  $i$ , and  $H_{i,j}(t)$  is the channel coefficient from transmitter  $j$  to receiver  $i$  over time slot  $t$ . We remove the time index in the rest of

the paper for brevity unless it is needed. The signals  $Y_i$  and  $X_i$  correspond to the receive and transmit signals at the  $i^{\text{th}}$  base station and mobile terminal in the uplink, respectively, and the  $i^{\text{th}}$  mobile terminal and base station in the downlink respectively. For consistency of notation, we will always refer to  $H_{i,j}$  as the channel coefficient between mobile terminal  $i$  and base station  $j$ .

#### A. Channel Model

We consider the following locally connected interference network. The mobile terminal with index  $i$  is connected to base stations  $\{i, i-1, \dots, i-L\}$ , except the first  $L$  mobile terminals, which are connected only to all the base stations with a similar or lower index. More precisely,

$$H_{i,j} = 0 \text{ iff } i \notin \{j, j+1, \dots, j+L\}, \forall i, j \in [K], \quad (2)$$

and all non-zero channel coefficients are drawn from a continuous joint distribution. Finally, we assume that global channel state information is available at all mobile terminals and base stations.

#### B. Cell Association

For each  $i \in [K]$ , let  $\mathcal{C}_i \subseteq [K]$  be the set of base stations, with which mobile terminal  $i$  is associated, i.e., those base stations that carry the terminal's message in the downlink and will have its decoded message for the uplink. The transmitters in  $\mathcal{C}_i$  cooperatively transmit the message (word)  $W_i$  to mobile terminal  $i$  in the downlink. In the uplink, one of the base station receivers in  $\mathcal{C}_i$  will decode  $M_i$  and pass it to the remaining receivers in the set. We consider a cell association constraint that bounds the cardinality of the set  $\mathcal{C}_i$  by a number  $N_c$ ; this constraint is one way to capture a limited backhaul capacity constraint where not all messages can be exchanged over the backhaul.

$$|\mathcal{C}_i| \leq N_c, \forall i \in [K]. \quad (3)$$

We would like to stress on the fact that we only allow full messages to be shared over the backhaul. More specifically, splitting messages into parts and sharing them as in [15], or sharing of quantized signals as in [9] is not allowed.

#### C. Degrees of Freedom

Let  $P$  be the average transmit power constraint at each transmitter, and let  $\mathcal{W}_i$  denote the alphabet for message  $W_i$ . Then the rates  $R_i(P) = \frac{\log |\mathcal{W}_i|}{n}$  are achievable if the decoding error probabilities of all messages can be simultaneously made arbitrarily small for a large enough coding block length  $n$ , and this holds for almost all channel realizations. The degrees of freedom  $d_i, i \in [K]$ , are defined as  $d_i = \lim_{P \rightarrow \infty} \frac{R_i(P)}{\log P}$ . The DoF region  $\mathcal{D}$  is the closure of the set of all achievable DoF tuples. The total number of degrees of freedom ( $\eta$ ) is the maximum value of the sum of the achievable degrees of freedom,  $\eta = \max_{\mathcal{D}} \sum_{i \in [K]} d_i$ .

For a  $K$ -user locally connected with connectivity parameter  $L$ , we define  $\eta(K, L, N_c)$  as the best achievable  $\eta$  on average

taken over both downlink and uplink sessions over all choices of transmit sets satisfying the backhaul load constraint in (3). In order to simplify our analysis, we define the asymptotic per user DoF (puDoF)  $\tau(L, N_c)$  to measure how  $\eta(K, L, N_c)$  scales with  $K$  while all other parameters are fixed,

$$\tau(L, N_c) = \lim_{K \rightarrow \infty} \frac{\eta(K, L, N_c)}{K}. \quad (4)$$

We further define  $\tau_D(L, N_c)$  and  $\tau_U(L, N_c)$  as the puDoF when we optimize only for the downlink and uplink session, respectively.

#### D. Zero-forcing (Interference Avoidance) Schemes

We consider in this work the class of *interference avoidance* schemes, where each message is either not transmitted or allocated one degree of freedom. Accordingly, every receiver is either active or inactive. An active receiver does not observe any interfering signals.

We add the superscript **zf** to the puDoF symbol when we impose the constraint that the coding scheme that can be used has to be a zero-forcing scheme. For example,  $\tau_U^{\text{zf}}(L, N_c)$  denotes the puDoF value when considering only the uplink and impose the restriction to zero-forcing schemes.

### III. PRIOR WORK: DOWNLINK-ONLY SCHEME

In [11], the considered setting was studied for only downlink transmission. When restricting our choice of coding scheme to zero-forcing schemes, the puDoF value was characterized as,

$$\tau_D^{\text{zf}}(L, N_c) = \frac{2N_c}{2N_c + L}, \quad (5)$$

and the achieving cell association was found to be the following. The network is split into subnetworks; each with consecutive  $2N_c + L$  transmitter-receiver pairs. The last  $L$  transmitters in each subnetwork are inactive to avoid inter-subnetwork interference. The zero-forcing scheme aims to deliver  $2N_c$  messages free of interference in each subnetwork, so that the achieved puDoF value is as in (5). In order to do that with a cooperation constraint that limits each message to be available at  $N_c$  transmitters, we create two Multiple Input Single Output (MISO) Broadcast Channels (BC) within each subnetwork; each with  $N_c$  transmitter-receiver pairs, and ensure that interference across these channels is eliminated. We now discuss the cell association in the first subnetwork, noting that the remaining subnetworks follow an analogous pattern. The first MISO BC consists of the first  $N_c$  transmitter-receiver pairs. For each  $i \in \{1, 2, \dots, N_c\}$ , message  $W_i$  is associated with base stations with indices in the following set,  $\mathcal{C}_i = \{i, i+1, \dots, N_c\}$ . The second MISO BC consists of the  $N_c$  transmitters with indices in the set  $\{N_c+1, N_c+2, \dots, 2N_c\}$  and the  $N_c$  receivers with indices in the set  $\{N_c+L+1, N_c+L+2, \dots, 2N_c+L\}$ . Note that the middle  $L$  receivers in each subnetwork are deactivated to eliminate interference between the two MISO BCs. For each  $i \in \{N_c+L+1, N_c+L+2, \dots, 2N_c+L\}$ , message  $W_i$  is associated with transmitters that have indices in the

set  $\mathcal{C}_i = \{i-L, i-L-1, \dots, N_c+1\}$ . It was shown in [11] that the puDoF value of (5) achieved by this scheme is that best achievable value in the downlink using the imposed cooperation constraint and zero-forcing schemes.

### IV. UPLINK-ONLY SCHEME

We discuss in this section backhaul designs that optimize only the uplink rate, and consider only zero-forcing coding schemes. We present the following inner bound on the puDoF value that is characterised by a piecewise function as follows:

$$\tau_U^{\text{zf}}(L, N_c) \geq \begin{cases} 1 & L+1 \leq N_c, \\ \frac{N_c+1}{L+2} & \frac{L}{2} \leq N_c \leq L, \\ \frac{2N_c}{2N_c+L} & 1 \leq N_c \leq \frac{L}{2} - 1. \end{cases} \quad (6)$$

The cell association that achieved the above is as following. When  $N_c \geq L+1$ , the optimal association is similar to the one specified in [13, Section 4], where each mobile terminal is associated with the  $L+1$  base stations connected to it. The last base station, with index  $K$ , in the network decodes the last message and then passes it on to the  $L$  other base stations connected to the  $K^{\text{th}}$  mobile terminal, eliminating all interference caused by that mobile terminal. Each base station then decodes its message and passes it on to the other base stations, eliminating the interference caused by the message. Thus, one degree of freedom is achieved for each user.

In the second range  $\frac{L}{2} \leq N_c \leq L$ , the cell association that achieves a puDoF value of  $\frac{N_c+1}{L+2}$  is as follows. The network is split into subnetworks, each with consecutive  $L+2$  transmitter-receiver pairs. In each subnetwork, we decode the last  $N_c+1$  words. For each  $i \in \{L+2, L+1, \dots, L+2-N_c+1\}$ , message  $W_i$  is associated with  $\{i, i-1, \dots, L+2-N_c+1\} \subseteq \mathcal{C}_i$ . Thus the last  $N_c$  words are decoded. The base stations with indices in the set  $\{2, 3, \dots, L+2-N_c\}$  are inactive as there is interference from the last transmitter in the subnetwork which cannot be eliminated. The first base station decodes  $W_{L+2-N_c}$ . To eliminate the interference caused by the transmitters in the set  $\mathcal{S} = \{L+2-N_c+1, L+2-N_c+2, \dots, L+1\}$  at the first base station of the subnetwork, we add the first base station to each  $\mathcal{C}_i, \forall i \in \mathcal{S}$ . Now for messages with indices in the set  $\mathcal{S}$ , we have used up  $\alpha_i = 2+i-(L+2-N_c+1)$  associations; the factor of two comes from the base station resolving  $W_i$  and the first base station of the subnetwork. But each transmitter with indices in the set  $\mathcal{S} \setminus \{L+1\}$  also interferes with the subnetwork directly preceding this subnetwork.  $\forall i \in \mathcal{S} \setminus \{L+1\}$ , the message  $W_i$  interferes with the bottom  $L+1-i$  base stations of the preceding subnetwork, which is precisely the number of associations left for the respective message i.e.  $N_c - \alpha_i = L+1-i$ , thus inter-subnetwork interference can be eliminated at those base stations.

In the third range  $1 \leq N_c \leq \frac{L}{2} - 1$ , the cell association that achieves the lower bound of  $\frac{2N_c}{2N_c+L}$  is similar to the one described in Section III for the downlink. The network is split into disjoint subnetworks; each with consecutive  $2N_c+L$  transmitter-receiver pairs. For the uplink, we consider two sets of indices for transmitters  $\mathcal{A}_T = \{1, 2, \dots, N_c\}$  and

$\mathcal{B}_T = \{N_c + L + 1, N_c + L + 2 \dots, 2N_c + L\}$ , and corresponding sets of receivers  $\mathcal{A}_R = \{1, 2, \dots, N_c\}$  and  $\mathcal{B}_R = \{N_c + 1, N_c + L + 2 \dots, 2N_c\}$ . For each  $i \in \mathcal{A}_T$ , the message  $W_i$  is associated with the receivers receiving it in  $\mathcal{A}_R$ . Receiver  $i$  decodes  $W_i$  and the other associations in  $\mathcal{C}_i$  exist for eliminating interference. Similarly For each  $j \in \mathcal{B}_T$ , the message  $W_j$  is associated with the receivers receiving it in  $\mathcal{B}_R$ , but now receiver  $j - L$  decodes  $W_j$  and the other associations in  $\mathcal{C}_j$  are for eliminating interference.

We observe that if we were not restricted to the zero-forcing coding scheme then for the third range, we could achieve  $\frac{1}{2}$  puDoF using the asymptotic interference alignment scheme of [16].

## V. PRELIMINARY CONVERSE PROOF WHEN $N_c = L$

In this section, we provide a converse proof for a special case of the second range of (6). When  $N_c = L$ , The optimal zero-forcing puDoF for the uplink can be characterised as:

$$\tau_U^{\text{zf}}(L, L) = \frac{L+1}{L+2}. \quad (7)$$

We begin by dividing the network into subnetworks of  $L+2$  consecutive transmitters-receiver pairs. We observe that in any subnetwork, if we have  $N_c + 1 = L + 1$  consecutive active receivers (base stations), then the transmitter connected to all these receivers must be inactive, because a message's interference cannot be canceled at  $N_c$  or more receivers. Let  $\Gamma_{BS}$  be the set of subnetworks where all  $N_c + 2$  receivers are active, and  $\Phi_{BS}$  be the set of subnetworks with at most  $N_c$  active receivers. Similarly, let  $\Gamma_{MT}$  and  $\Phi_{MT}$  be the subnetworks with  $N_c + 2$  active transmitters and at most  $N_c$  active transmitters, with respect to order. To be able to achieve a higher puDoF than (7), it must be true that both conditions hold:  $|\Gamma_{BS}| > |\Phi_{BS}|$  and  $|\Gamma_{MT}| > |\Phi_{MT}|$ . Now note that for any subnetwork that belongs to  $\Gamma_{BS}$ , at most  $N_c$  transmitters will be active, because the interference caused by any message cannot be canceled at  $N_c$  or more receivers. Hence  $\Gamma_{BS} \subseteq \Phi_{MT}$ . Further, the same logic applies to conclude that for any subnetwork with  $N_c + 1$  active receivers, the number of active transmitters is at most  $N_c + 1$ , and hence  $\Gamma_{MT} \subseteq \Phi_{BS}$ . It follows that if  $|\Gamma_{BS}| > |\Phi_{BS}|$ , then  $|\Gamma_{MT}| < |\Phi_{MT}|$ , and hence the statement is proved.

We conjecture that a similar argument holds to extend (7) and prove that  $\tau_U^{\text{zf}}(L, N_c) = \frac{N_c+1}{L+2}$  for the whole second range of (6), i.e., when  $\frac{L}{2} \leq N_c \leq L$ .

## VI. AVERAGE UPLINK-DOWNLINK DEGREES OF FREEDOM

In [13], the puDoF value  $\tau(L=1, N_c)$  was characterized. Here, we present zero-forcing schemes, with the goal of optimizing the average rate across both uplink and downlink for arbitrary values of  $L \geq 2$ . The corresponding puDoF inner bounds are given by,

$$\tau^{\text{zf}}(L, N_c) \geq \begin{cases} \frac{1}{2} \left( 1 + \left( \frac{\lceil \frac{L}{2} \rceil + \delta + N_c - (L+1)}{N_c} \right) \right) & L+1 \leq N_c, \\ \frac{2N_c}{2N_c+L} & 1 \leq N_c \leq L, \end{cases} \quad (8)$$

where  $\delta = (L+1) \bmod 2$ .

The coding scheme that achieves the inner bound for the second range of (8) is essentially the union of the scheme described in Section III and the scheme that achieves the third range of (6). The network is split into disjoint subnetworks; each with consecutive  $2N_c + L$  transmitter-receiver pairs. We consider two sets of base stations  $\mathcal{A}_{BS} = \{1, 2, \dots, N_c\}$  and  $\mathcal{B}_{BS} = \{N_c + 1, N_c + L + 2 \dots, 2N_c\}$ , and two sets of mobile terminals  $\mathcal{A}_{MT} = \{1, 2, \dots, N_c\}$  and  $\mathcal{B}_{MT} = \{N_c + L + 1, N_c + L + 2 \dots, 2N_c + L\}$ . Now for each  $i \in \mathcal{A}_{MT}$ ,  $\mathcal{C}_i = \mathcal{A}_{BS}$ . Similarly for each  $j \in \mathcal{B}_{MT}$ ,  $\mathcal{C}_j = \mathcal{B}_{BS}$ . Thus, for the downlink, we can get the optimal puDoF described in Section III, and for the uplink, we can get the inner bound stated in the third range of (6).

For the case where  $N_c \geq L + 1$ , the coding scheme that achieves the expression described in (8) is as follows. First, we associate each mobile terminal with the  $L + 1$  base stations connected to it. This achieves the puDoF value of unity during the uplink in the same way as the scheme that achieves it in Section IV. Hence, we know so far that  $\mathcal{C}_i \supseteq \{i, i-1, i-2, \dots, i-L\} \cap [K], \forall i \in [K]$ . During the downlink, we divide the network into disjoint subnetworks; each consists of  $N_c$  consecutive transmitter-receiver pairs. This allows us to create in each subnetwork a MISO broadcast channel. Let  $\chi$  be the number of transmitter-receiver pairs with an inactive node between the last active base station of one sub-network and the first active mobile terminal in the following subnetwork. Then we observe that in order to eliminate inter-subnetwork interference, it has to be the case that  $\chi \geq L$ . Because the achieved DoF in any subnetwork is bound by the minimum of the number of active transmitters and the number of active receivers in the subnetwork, we set the number of inactive mobile terminals to be the same as the number of inactive base stations. Let that aforementioned number be  $\epsilon$ , then  $2\epsilon = \chi \geq L$ . Since minimizing  $\epsilon$  will maximize the achieved DoF, we set  $\epsilon = \lceil \frac{L}{2} \rceil$ . As we are leaving the first  $\epsilon$  mobile terminals inactive in the subnetwork, the first base station (call it BS  $p$ ) will be transmitting message  $W_{p+\epsilon}$  to the the mobile terminal MT  $p + \epsilon$ . For this broadcast channel to work, each active base station must be associated with all active mobile terminals in the subnetwork, so that all interfering signals can be eliminated at each mobile terminal receiver.

When  $\delta = 0$ , BS  $p + \epsilon$  will be delivering  $W_{p+L+1}$ , whose mobile terminal was not be associated with BS  $p$  through the uplink assignment, so we will need to add  $p$  to  $\mathcal{C}_{p+L+1}$ . Thus we can only have  $N_c - (L + 1) - 1$  active base stations, among the base stations in the subnetwork that have indices greater than  $p + \epsilon$ . This is because if BS  $j$ , where  $j \geq p + \epsilon + N_c - (L + 1)$ , is active then to ensure that the interference caused by it does not propagate, we have to have  $p \in \mathcal{C}_{j+\epsilon}$ , but then  $|\mathcal{C}_{j+\epsilon}| > N_c$ , i.e., mobile terminal  $j + \epsilon$  has to be associated with more base stations than what the backhaul constraint allows for. So in each subnetwork, we will have a total of  $\Delta = \epsilon + N_c - (L + 1)$  words transmitted without interference, out of a total of  $\Delta + \epsilon = N_c$  mobile terminals in

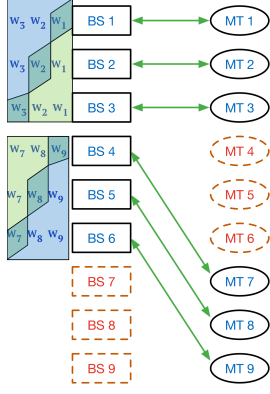


Fig. 1: Scheme for average uplink (blue shade) and downlink (green shade) communication when  $N_c \leq L$

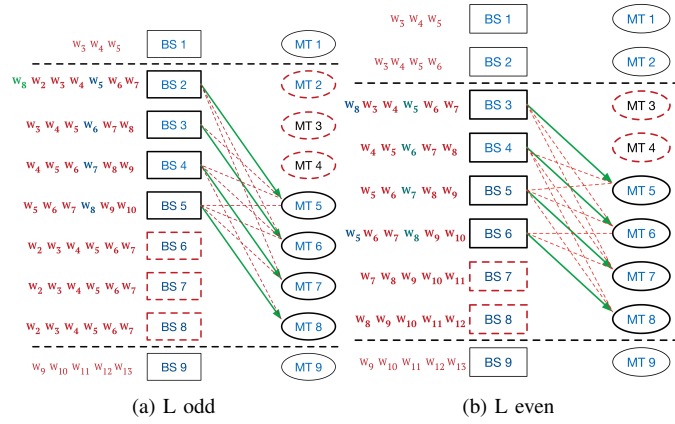


Fig. 2: Scheme for downlink, with all the associations needed for optimal uplink, that achieves the lower bound defined in equation (8) when  $N_c \geq L + 1$

the subnetwork.

When  $\delta = 1$ , BS  $p + \epsilon + 1$  will be delivering  $W_{p+L+1}$ , but to ensure that the mobile terminals connected to this base station other than MT  $p + L + 1$  do not suffer from interference, we need to add  $p$  to  $\mathcal{C}_{p+L+1}$  and  $p + \epsilon + 1$  to  $\mathcal{C}_{p+\epsilon}$ . Thus we can only have  $N_c - (L + 1)$  active base stations among the base stations whose indices are greater than  $p + \epsilon$  in the subnetwork. Otherwise, if BS  $j'$ , where  $j' \geq p + \epsilon + N_c - L$ , was active then we need to have  $j' \in \mathcal{C}_{p+\epsilon}$ , which results in  $|\mathcal{C}_{p+\epsilon}| = N_c + 1 > N_c$ . So in each subnetwork we will have a total of  $\Delta = \epsilon + \delta + N_c - (L + 1)$  words transmitted without interference, out of a total of  $\Delta + \epsilon = N_c$  words. Figures 1 and 2 serve as examples for the above scheme. Figure 1 uses values of  $N_c = 3$ , and  $L = 3$ . Using this scheme we get a puDoF of  $\frac{2}{3}$ , which is equivalent to  $\frac{2N_c}{2N_c + L}$ . Figure 2 uses values of  $N_c = L + 2$  for  $L = 5$  and  $L = 4$  for parts a and b, respectively. The achieved puDoF values are  $\frac{4}{7}$  or  $\frac{4}{6}$ , respectively.

## VII. CONCLUDING REMARKS

It is important to note that most of the presented decoding schemes for the uplink require a propagation delay that scales with the number of users in the network  $K$ , even if the employed subnetworks are small. We plan to consider delay constraints in future work. We also would like to highlight that this theoretical work is a preliminary effort to understand optimal cell association decisions in cellular networks, with cloud-based controllers that access the base stations through a limited backhaul. Both channel model and cooperation constraint assumptions may not be practical. Nevertheless, we believe that the captured insights can lead to rigorous solutions for more practical settings.

## REFERENCES

- [1] S. Veetil, K. Kuchi and R. K. Ganti. (2015, Dec.). Performance of cloud radio access networks. [Online]. Available: <http://arxiv.org/pdf/1512.05904v1.pdf>
- [2] A. Checko, H. L. Christiansen, Y. Yan, L. Scolari, G. Kardaras, M. S. Berger and L. Dittmann, "Cloud RAN for mobile networks - a technology overview," *IEEE Communication Surveys Tutorials*, vol. 17, no. 1, pp. 405-426, First Quart. 2015.
- [3] China Mobile, "Next generation fronthaul interface," White Paper, Oct. 2015.
- [4] The 5G Infrastructure Public Private Partnership. (2015, Jan.). 5G-Xhaul Project. [Online]. Available: <https://5g-ppp.eu/5g-xhaul/>
- [5] O. Simeone, A. Maeder, M. Peng, O. Sahin and W. Yu. (2015, Dec.). Cloud radio access network: Virtualizing wireless access for dense heterogeneous systems. [Online]. Available: <http://arxiv.org/abs/1512.07743>.
- [6] S. -H. Park, O. Simeone and S. Shamai. (2016, Jan.). Joint optimization of cloud and edge processing for fog radio access networks. [Online]. Available: <http://arxiv.org/abs/1601.02460>.
- [7] P. Marsch and G. P. Fettweis, *Coordinated Multi-Point in Mobile Communications: From Theory to Practice*, 1st ed. New York, NY: Cambridge, 2011.
- [8] V. S. Annapureddy, A. El Gamal, V. V. Veeravalli, "Degrees of Freedom of Interference Channels with CoMP Transmission and Reception," *IEEE Trans. Inf. Theory*, vol. 58, no. 9, pp. 5740-5760, Sep. 2012.
- [9] V. Ntranos, M. Maddah-Ali, G. Caire. (2014, Jul.). On uplink-downlink duality for cellular IA. [Online]. Available: <https://arxiv.org/abs/1407.3538>.
- [10] M. H. M. Costa, "Writing on dirty paper (corresp.)," *IEEE Trans. Inf. Theory*, vol. 29, pp. 439-441, May 1983.
- [11] A. El Gamal, V. S. Annapureddy, and V. V. Veeravalli, "Interference channels with coordinated multi-point transmission: Degrees of freedom, message assignment, and fractional reuse", *IEEE Trans. Inf. Theory*, vol. 60, no. 6, pp. 3483-3498, Mar. 2014.
- [12] M. Bande, A. El Gamal, V. V. Veeravalli. (2016, Oct.). Degrees of Freedom in Wireless Interference Networks with Cooperative Transmission and Backhaul Load Constraints. [Online]. Available: <https://arxiv.org/abs/1610.09453>.
- [13] A. El Gamal, "Cell associations that maximize the average uplink-downlink degrees of freedom," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, Barcelona, Spain, Jul. 2016.
- [14] A. Wyner, "Shannon-theoretic approach to a Gaussian cellular multiple-access channel," *IEEE Trans. Inf. Theory*, vol. 40, no. 5, pp. 1713-1727, Nov. 1994.
- [15] M. Wigger, R. Timo and S. Shamai (2016, Mar.). Conferencing in Wyner's Asymmetric Interference Network: Effect of Number of Rounds. [Online]. Available: <http://arxiv.org/abs/1603.05540>
- [16] V. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425-3441, Aug. 2008.